PreCalc — Quadratic Functions and Models

Pg. 163 #1-18, 21, 31, 41, 51, 55, 57, 63, 67, 71, 73, 77, 79, 83, 87, 99, 101 — Section 3.1

1. $(0,-9), (-3,0), (3,0)$

2. $2x^2+7x-4=0$

$(2x-1)(x+4)=0$

$2x-1=0$ or $x+4=0$

$2x=1$ $\qquad x=-4$

$x=\frac{1}{2}$

The solution set is $\left\{-4, \frac{1}{2}\right\}$.

3. $\frac{25}{4}$

4. right; 4

5. parabola

6. axis of symmetry

7. $-\frac{b}{2a}$

8. True; $a=2>0$.

9. True; $-\frac{b}{2a}=-\frac{4}{2(-1)}=2$

10. True

11. C

12. E

13. F

14. A

15. G

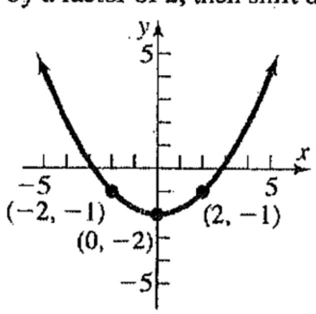
16. B

17. H

18. D

21. $f(x)=\frac{1}{4}x^2-2$

Using the graph of $y=x^2$, compress vertically by a factor of 2, then shift down 2 units.



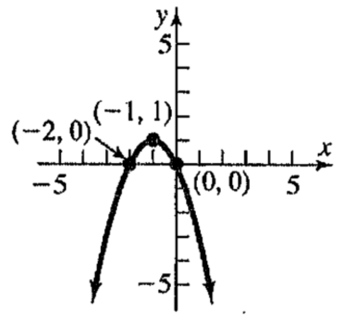
31. $f(x)=-x^2-2x$

$=-\left(x^2+2x\right)$

$=-(x^2+2x+1)+1$

$=-(x+1)^2+1$

Using the graph of $y=x^2$, shift left 1 unit, reflect across the x-axis, then shift up 1 unit.



41. For $f(x)=x^2+2x-8$, $a=1$, $b=2$, $c=-8$.

Since $a=1>0$, the graph opens up.

The x-coordinate of the vertex is

$$x=\frac{-b}{2a}=\frac{-2}{2(1)}=\frac{-2}{2}=-1.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right)=f(-1)$$

$$=(-1)^2+2(-1)-8=1-2-8=-9.$$

Thus, the vertex is $(-1,-9)$.

The axis of symmetry is the line $x=-1$.

The discriminant is:

$b^2-4ac=2^2-4(1)(-8)=4+32=36>0$,

so the graph has two x-intercepts.

The x-intercepts are found by solving:

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=-4$ or $x=2$.

The x-intercepts are -4 and 2.

The y-intercept is $f(0)=-8$.

51. For $f(x)=-4x^2-6x+2$, $a=-4$, $b=-6$, $c=2$. Since $a=-4<0$, the graph opens down. The x-coordinate of the vertex is

$$x=\frac{-b}{2a}=\frac{-(-6)}{2(-4)}=\frac{6}{-8}=-\frac{3}{4}.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right)=f\left(-\frac{3}{4}\right)=-4\left(-\frac{3}{4}\right)^2-6\left(-\frac{3}{4}\right)+2$$
$$=-\frac{9}{4}+\frac{9}{2}+2=\frac{17}{4}.$$

Thus, the vertex is $\left(-\frac{3}{4},\frac{17}{4}\right)$.

The axis of symmetry is the line $x=-\frac{3}{4}$.

The discriminant is:

$$b^2-4ac=(-6)^2-4(-4)(2)=36+32=68,$$

so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$-4x^2-6x+2=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-6)\pm\sqrt{68}}{2(-4)}$$
$$=\frac{6\pm\sqrt{68}}{-8}=\frac{6\pm2\sqrt{17}}{-8}$$
$$=\frac{3\pm\sqrt{17}}{-4}$$

The x-intercepts are $\frac{-3+\sqrt{17}}{4}$ and $\frac{-3-\sqrt{17}}{4}$.

55. Since it is given that the graph passes through $(0,-4)$, then $f(0)=-4$.

So $f(x)=ax^2+bx+c$

$$-4=a(0)^2+b(0)+c$$
$$-4=0+0+c$$
$$c=-4.$$

Since it is given that the vertex is at $(-3,5)$, then

$f(-3)=5$ and $\frac{-b}{2a}=-3\Rightarrow b=6a$.

So $f(x)=ax^2+bx-4$

$$5=a(-3)^2+b(-3)-4$$
$$5=9a-3b-4$$
$$9=9a-3b$$
$$9=9a-3(6a)$$
$$9=9a-18a$$
$$9=-9a$$
$$a=-1.$$

So, $b=6a=6(-1)=-6$.

Therefore, $f(x)=ax^2+bx+c=-x^2-6x-4$.

57. Since it is given that the graph passes through $(3,5)$, then $f(3)=5$.

So $f(x)=ax^2+bx+c$

$$5=a(3)^2+b(3)+c$$
$$5=9a+3b+c$$

Since it is given that the vertex is at $(1,-3)$, then

$f(1)=-3$ and $\frac{-b}{2a}=1 \Rightarrow b=-2a$.

So $f(x)=ax^2+bx+c$

$$-3=a(1)^2+b(1)+c$$
$$-3=a+b+c$$

Since $5=9a+3b+c$ and $-3=a+b+c$, then

$8=8a+2b$

$8=8a+2(-2a)$

$8=8a-4a$

$8=4a$

$a=2.$

Then $b=-2a=-2(2)=-4$ and

$-3=a+b+c$

$-3=2+(-4)+c$

$-3=-2+c$

$c=-1.$

Therefore, $f(x)=ax^2+bx+c=2x^2-4x-1$.

63. For $f(x)=-x^2+10x-4$, $a=-1$, $b=10$, $c=-4$. Since $a=-1<0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x=\frac{-b}{2a}=\frac{-10}{2(-1)}=\frac{-10}{-2}=5$.

The maximum value is

$$f\left(\frac{-b}{2a}\right)=f(5)=-(5)^2+10(5)-4.$$
$$=-25+50-4$$
$$=21.$$

X	Y_1	
3	17	
4	20	
5	21	
6	20	
7	17	
8	12	
9	5	

X=5

67. Use the form $f(x)=a(x-h)^2+k$.

The vertex is $(0,2)$, so $h=0$ and $k=2$.

$$f(x)=a(x-0)^2+2=ax^2+2.$$

Since the graph passes through $(1,8)$, $f(1)=8$.

$f(x)=ax^2+2$

$8=a(1)^2+2$

$8=a+2$

$6=a$

$f(x)=6x^2+2.$

$a=6, b=0, c=2$

71. $R(p)=-4p^2+4000p$, $a=-4, b=4000, c=0.$

Since $a=-4<0$, the graph is a parabola that opens down, so the vertex is a maximum point. The maximum occurs at

$$p=\frac{-b}{2a}=\frac{-4000}{2(-4)}=500.$$

Thus, the unit price should be \$500 for maximum revenue. The maximum revenue is

$$R(500)=-4(500)^2+4000(500)$$
$$=-1000000+2000000$$
$$=\$1{,}000{,}000.$$

73. a. $R(x)=x\left(-\frac{1}{6}x+100\right)=-\frac{1}{6}x^2+100x$

b. $R(200)=-\frac{1}{6}(200)^2+100(200)$

$=\frac{-20000}{3}+20000$

$=\frac{40000}{3}\approx \$13,333.33$

c. $x=\frac{-b}{2a}=\frac{-100}{2\left(-\frac{1}{6}\right)}=\frac{-100}{\left(-\frac{1}{3}\right)}=\frac{300}{1}=300$

The maximum revenue is

$R(300)=-\frac{1}{6}(300)^2+100(300)$

$=-15000+30000$

$=\$15,000$

d. $p=-\frac{1}{6}(300)+100=-50+100=\$50$

77. a. Let $x=$ width and $y=$ length of the rectangular area.

Solving $P=2x+2y=400$ for y:

$y=\frac{400-2x}{2}=200-x$.

Then $A(x)=(200-x)x=200x-x^2$.

$=-x^2+200x.$

b. $x=\frac{-b}{2a}=\frac{-200}{2(-1)}=\frac{-200}{-2}=100$ yards

c. $A(100)=-100^2+200(100)$

$=-10000+20000$

$=10,000$ sq yds.

79. Let $x=$ width and $y=$ length of the rectangular area. Solving $P=2x+y=4000$ for y:

$y=4000-2x$. Then

$A(x)=(4000-2x)x=4000x-2x^2=-2x^2+4000x$

$x=\frac{-b}{2a}=\frac{-4000}{2(-2)}=\frac{-4000}{-4}=1000$ meters

maximizes area.

$A(1000)=-2(1000)^2+4000(1000)$.

$=-2000000+4000000$

$=2,000,000$

The largest area that can be enclosed is 2,000,000 square meters.

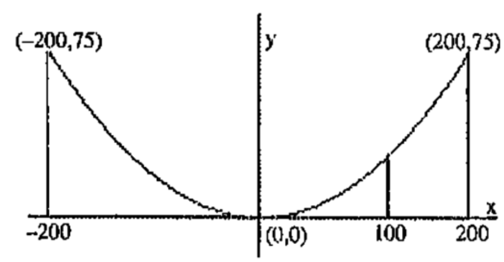
83. Locate the origin at the point where the cable touches the road. Then the equation of the parabola is of the form: $y=ax^2$, where $a>0$. Since the point (200, 75) is on the parabola, we can find the constant a:

Since $75=a(200)^2$, then

$a=\frac{75}{200^2}=0.001875$.

When $x=100$, we have:

$y=0.001875(100)^2=18.75$ meters.



87. Let $x =$ the width of the rectangle or the diameter of the semicircle and let $y =$ the length of the rectangle.

The perimeter of each semicircle is $\frac{\pi x}{2}$.

The perimeter of the track is given by: $\frac{\pi x}{2} + \frac{\pi x}{2} + y + y = 1500$.

Solving for x:

$$\pi x + 2y = 1500$$
$$\pi x = 1500 - 2y$$
$$x = \frac{1500 - 2y}{\pi}$$

The area of the rectangle is:

$$A = xy = \left(\frac{1500 - 2y}{\pi}\right) y = \frac{-2}{\pi} y^2 + \frac{1500}{\pi} y.$$

This equation is a parabola opening down; thus, it has a maximum when

$$y = \frac{-b}{2a} = \frac{\frac{-1500}{\pi}}{2\left(\frac{-2}{\pi}\right)} = \frac{-1500}{-4} = 375..$$

Thus, $x = \frac{1500 - 2(375)}{\pi} = \frac{750}{\pi} \approx 238.73$

The dimensions for the rectangle with maximum area are $\frac{750}{\pi} \approx 238.73$ meters by 375 meters.

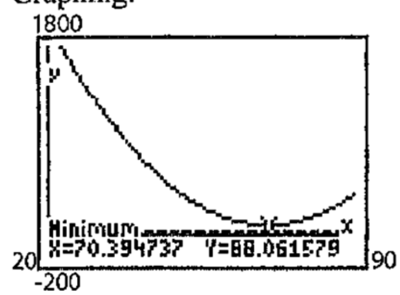
99. **a.** $M(23) = .76(23)^2 - 107.00(23) + 3854.18$
$= 1795.22$ victims.

b. Solve for x:

$$M(x) = 0.76x^2 - 107.00x + 3854.18 = 1456$$
$$0.46x^2 - 107.00x + 3854.18 = 1456$$
$$0.76x^2 - 107.00x + 2398.18 = 0$$
$$a = 0.76, b = -107.00, c = 2398.18$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-107) \pm \sqrt{(-107)^2 - 4(0.76)(2398.18)}}{2(0.76)}$$
$$= \frac{107 \pm \sqrt{4158.5328}}{1.52}$$
$$\approx \frac{107 \pm 64.49}{1.52} \approx 112.82 \text{ or } 27.97$$

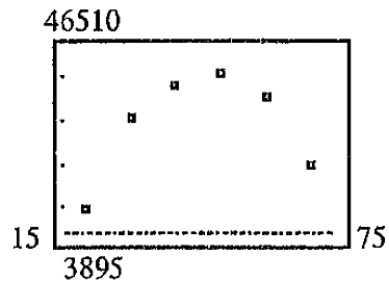
Since the model is valid on the interval $20 \le x < 90$, the only solution is $x \approx 27.97$ years of age.

c. Graphing:

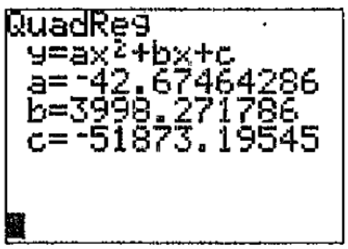


d. As age increases between the ages of 20 and 70.39, the number of murder victims decreases. After age 70.39, the number of murder victims increases as age increases

101. a. Graphing:The data appear to be quadratic with $a < 0$.



b. Using the QUADratic REGression program



$I(x) = -42.67x^2 + 3998.27x - 51873.2$

c. $x = \frac{-b}{2a} = \frac{-3998.27}{2(-42.67)} \approx 46.85$

An individual will earn the most income at an age of 46.85 years.

d. The maximum income will be:

$I(46.85)$

$= -42.67(46.85)^2 + 3998.27(46.85) - 51873.2$

$\approx \$41,788.41$

e.